Ultimate Strength Analysis of Aircraft Structures

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The results of ultimate static strength tests from different types of aircraft structures and structural parts obtained from several aircraft manufacturers were statistically analyzed. By using test samples with at least 3 replications and reducing sample data to their mean, all results could be unified in a single population of over 300 data points and these points fitted by the third asymptotic distribution of smallest values (Weibull distribution). This distribution is used as a representative distribution of the ultimate strength of an aircraft combined with the ratio between the design ultimate load and the ultimate strength attained in actual tests, derived from the test data. By combining the distribution of strength with representative distributions of gusts in flight through thunderstorm turbulence and of operational loads, respectively, realistic reliability functions for ultimate load failure of gust-sensitive (long-range) and of maneuver-sensitive (short-range) aircraft structures were obtained for various assumed levels of the ultimate design load.

1. Introduction

IT is the purpose of this study to estimate the structural reliability of critical parts of airframes on the basis of ultimate strength test data of aircraft structures from various sources and of the spectrum of extreme gust and maneuver loads. The third asymptotic distribution function of extreme (smallest) values (Weibull distribution) has been chosen to fit the ultimate strength data obtained from tests on aircraft structural parts. Thus, the strength of any one member of the airframe can be expressed in terms of its design ultimate load with the aid of this distribution function.

A comparison is made between the recently obtained test data and the test results obtained by the Air Force about 20 years ago.[1] This comparison throws some light on certain aspects of aircraft structural development during this period. Representative thunderstorm and flight load spectra are adopted to match the distribution function of ultimate strength, and the risk of "ultimate load failure" is computed according to a standard procedure developed for this purpose[2] by assuming various values of the design gust or load factor. The associated reliability functions are subsequently evaluated in terms of the number of load application or of flight time of the aircraft.

2. Analysis of Data

Test data from 19 different types of structures and 38 types of panels have been obtained from various aircraft manufacturers through the efforts of the Air Force Materials Laboratory, Air Force Systems Command, Wright Patterson Air Force Base. These data have been analyzed and evaluated according to types of loading, types of structure and number of tests in each group and are summarized in Tables 1 and 2.

The expedient assumption is now made that the distribution of the ulimate strength of the test specimens can be considered to represent a single population, irrespective of the type of structures tested and its mode of failure, as long as this failure can be classified as "ultimate." This assumption is unavoidable because replications of ultimate load tests of large structures and structural parts are and will always be severely limited by technical and economic considerations. Without it, reliability analysis of aircraft structures becomes obviously impossible since the individual small samples are useless for this purpose.

Many distribution functions have been tried to fit the experimental results, but it appears that the Weibull distribution function provides a reasonably satisfactory representation of the data. The probability of survival expressed in terms of this distribution is given by the expression

$$L_X(x) = \exp[-(x/v)^k] \quad (1)$$

where by definition $L_X(x)$ is the probability of survival $L_X(x) = 1 - P_X(x)$, the variate $X = R_i/\bar{R}_i$, R_i being the ultimate strength of any one member in the ith group while $\bar{R}_i$ is the group mean of the ith group; v is the characteristic value of the distribution and k a scale factor.

The values of X-$(R_i/\bar{R}_i)$ have been calculated for every specimen in each group (at least 3 data values from nominally

Table 1 Data on ultimate strength of structures

Symbol[a]	Type of specimen	Type of loading	No. of tests
	Typhoon tailplane: semispan	Bending	14
	Typhoon tailplane modified: semispan	Bending	19
	Hudson tailplane: semispan	Bending	6
S_1	Whitley tailplane: semispan	Downward bending	13
	Whitley tailplane: semispan	Upward bending	7
S_2	Whitley tailplane: semispan	Torsion	21
	Mustang wings: asymmetric	Bending	5
S_3	F-80 tailplane	Bending	7
S_4	F-86D tailplane	Bending	3
	M.I.T. results {specimen type 1		3
	M.I.T. results {specimen type 2	Bending	3
	Box-beam tests	Bending	9
	F-51H wings[b]	Bending	3
	B-70 spar-skin composite beam[b]	Compression	3
S_5	C-130A, C-130B, C-130E wings[b]	Bending	10
	C-130A fuselage cabin (nose section)[b]	Internal pressure	4
	A-26B wings[b]	Bending	7
	A3D-2P wings[b]	Bending	3
S_6	B-58 sandwich box beam	Bending	5
			145 (19 groups)

[a] For extreme data points only.
[b] Data expressed in terms of *DUL* or *LD*.

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Table 2 Data on ultimate strength of components

Symbol[a]		Type of specimen	Type of loading	No. of tests	
C_1	Convair	Wing lower-surface plate stringer, *J* and *L* type stiffener	Compression	15	
		Wing lower-surface plate stringer, *J* type stiffener	Compression	7	
		Wing upper-surface plate stringer, stringer type "A"	Compression	12	
		Wing upper-surface plate stringer, stringer type "B"	Compression	6	
		Wing upper-surface plate stringer, stringer type "C"	Compression	6	
		Wing upper-surface plate stringer, stringer type "D"	Compression	6	
C_2	Douglas	Fuselage frame type "A"	Compression	4	
		Fuselage frame type "B"	Compression	4	
		Fuselage compression panel (DC-8)	Compression	8	
			Compression	3	
		Wing compression panel (DC-8)	Compression	3	
			Compression	3	
			Compression	3	
		Wing compression panel (DC-9)	Compression	6	
			Compression	6	
		Wing compression panel (DC-8, DC-9)	Compression	9	
		Fuselage shear panel (DC-8)	Shear	4	
			Shear	5	
		Fuselage shear panel (DC-9)	Shear	4	
			Shear	3	
			Shear	3	
C_3	Lockheed[b]	C-130 wing panel	Compression	6	
		C-130E, C-130B wing panel	Compression	8	
C_4	Northrop	YT-38 vertical stabilizer panel	Compression	4	
		YT-38 vertical stabilizer panel	Compression	4	
C_5	General Dynamics	Aluminum inner skin panel	Compression	5	54
		Shear panel	Shear	5	
C_6			Compression	4	
		Beaded aluminum panel	Compression	4	
C_7			Compression	11	
			Compression	4	
			Compression	3	
C_8		Aluminum beaded inner skin panel	Compression	3	
			Compression	3	
C_9			Compression	3	
		Sandwich and beaded skin panel	Compression	3	
			Compression	3	
C_{10}		Wing sandwich panel	Shear	3	
				196 (38 groups)	

[a] For extreme data points only.
[b] Data expressed in terms of *DUL* or *LD*.

identical specimens under the same type of loading in each group) and arranged in descending order of magnitude. These values have been plotted against the plotting position, $L_m = 1 - m/(n + 1)$ where $m = 1, 2, \ldots, n$ and n is the total number of data points, on extreme value probability paper as shown in Fig. 1.‡

A straight line has been drawn to fit the data points disregarding extreme points of high strength on the assumption that the required distribution of ultimate strength should be representative towards the low rather than towards the high range of data points. The extreme points at both tails of the distribution have been identified by letters (as listed in Tables 1 and 2).

The parameters of Eq. (1) are obtained graphically: $k = 31.0$ and $v = 1.014$. Thus, the probability of survival

$$L_X(x) = \exp[-(x/1.014)^{31}] \tag{2}$$

Some of the experimental data are expressed in terms of "design ultimate load" (design ultimate load = 1.5 limit load); these groups are identified by asterisks in Tables 1 and 2. As a result the group means of these groups can also be expressed in terms of their design ultimate load. The values of these group means have been plotted against $m/(n + 1)$ as shown in Fig. 2. A smooth curve has been chosen to fit these points fairly well, except for one extreme point of high value. The equation of this curve is

$$L_Y(y) = \exp[-(y/0.96)^{24}] \tag{3}$$

where $Y = \bar{R}_i/R_{DU_i}$ (R_{DU} = design ultimate load). It is assumed that the inclusion of the test data for which the design ultimate load or limit load has not been specified would not change the form and only insignificantly change the parameters of Eq. (3).

From Eqs. (2) and (3), the distribution and density functions of X and Y are easily obtained.

$$\begin{aligned} F_X(x) &= 1 - L_X(x) = 1 - \exp[-(x/1.014)^{31}] \\ F_Y(y) &= 1 - L_Y(y) = 1 - \exp[-(y/0.96)^{24}] \end{aligned} \tag{4}$$

Since $R_i = X\bar{R}_i$ and $\bar{R}_i = YR_{DU_i}$, it follows that

$$R_i = XYR_{DU_i} = ZR_{DU_i} \tag{5}$$

if $Z = XY$. Z is a random variable and R_{DU} the computed design value for a specific critical member of the aircraft structure. The distribution of Z represents therefore the distribution of ultimate strength of critical members of the airframe and thus of the airframes themselves. By definition,

$$F_Z(z) = P\{Z \leq z\} = P\{XY \leq z\} = \int_D\int f_{XY}(x,y)dxdy = \int_D\int f_X(x)f_Y(y)dxdy \tag{6}$$

‡ Data from one particular manufacturer are not included since they became known that in spite of nominal identity of tested specimens certain significant parameters were changed in the production process.

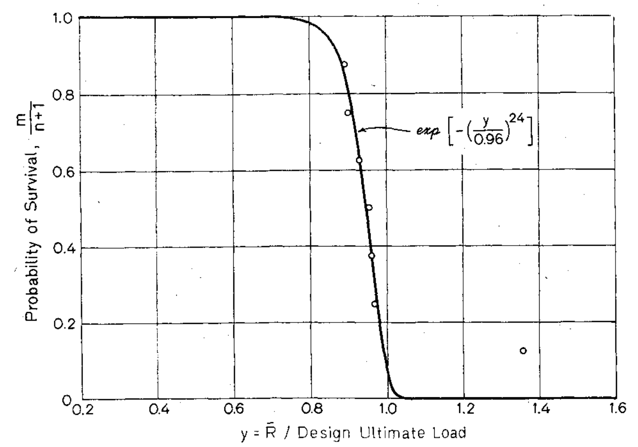

Fig. 1 Probability distribution of structural resistance (287 test data).

where D is the domain of integration below $z = xy$, hence,

$$F_Z(z) = \int_0^\infty f_X(x) \int_0^{z/x} f_Y(y)dydx = \int_0^\infty f_X(x)F_Y\left(\frac{z}{x}\right)dx$$

or

$$F_Z(z) = 1 - \int_0^\infty \left(\frac{31}{1.014}\right)\left(\frac{x}{1.014}\right)^{30} \exp\left[-\left(\frac{x}{1.014}\right)^{31}\right] \times \exp\left[-\left(\frac{z}{0.96x}\right)^{24}\right]dx \quad (7)$$

if the expressions of $f_X(x)$ and $F_Y(y)$ are substituted into the previous integral.

With the abbreviation, $t = \exp[-x/1.014)^{31}]$, Eq. (7) is transformed into

$$F_Z(z) = 1 - \int_0^1 \exp\left[-\left(\frac{z}{0.9734}\right)^{24}(-\ln t)^{-24/31}\right]dt \quad (8)$$

which is a form convenient for numerical evaluation. The associated reliability function

$$L_Z(z) = \int_0^1 \exp\left[-\left(\frac{z}{0.9734}\right)^{24}(-\ln t)^{-24/31}\right]dt \quad (9)$$

is presented in Fig. 3 (solid curve), for all values of Z; this curve can be fitted by the equation

$$L_Z(z) = \exp[-(z/0.96)^{19}] \quad (10)$$

The small difference between Eqs. (9) and (10) around $Z = 1.0$ is shown in Fig. 3.

The mean value and the variance of the random variable Z are[3] $EZ = v\Gamma(1 + 1/k)$ and $\mathrm{var}Z = v^2\{\Gamma(1 + 2/k) - [\Gamma(1 + 1/k)]^2\}$.

From Eq. (10), $v = 0.96$ and $k = 19$; therefore $EZ = 0.933$ and $\sigma_Z = (\mathrm{var}\ Z)^{1/2} = 0.061$. These values suggest that, in general, the mean value of the ultimate strength of nominally identical members of aircraft structures is about 93% of its nominal design ultimate load, while the standard deviation is about 6% of the design ultimate load.§

All test data expressed in terms of design ultimate load or limit load have also been considered without regard to the requirement of at least 3 test replications in each group and listed in Table 3. These data have been plotted in Fig. 4 and compared with the results of Jablecki's tests[1] performed

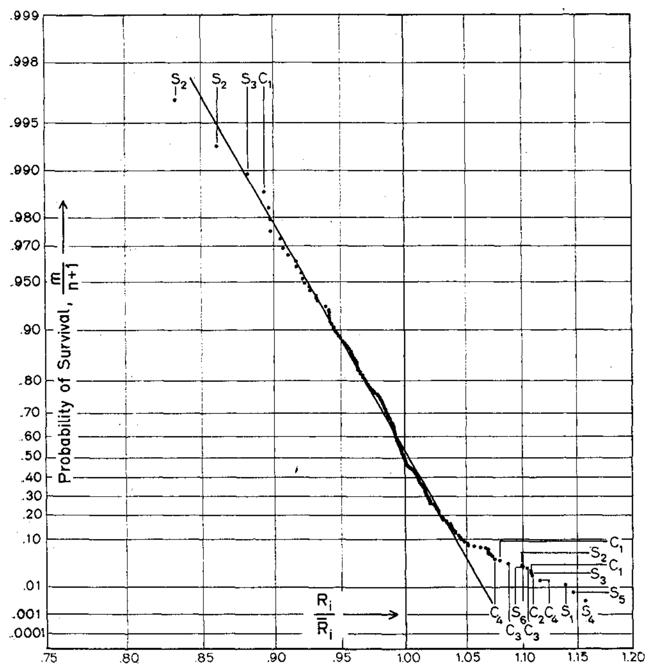

Fig. 2 Probability distribution of group means of structural resistance (based on 7 groups).

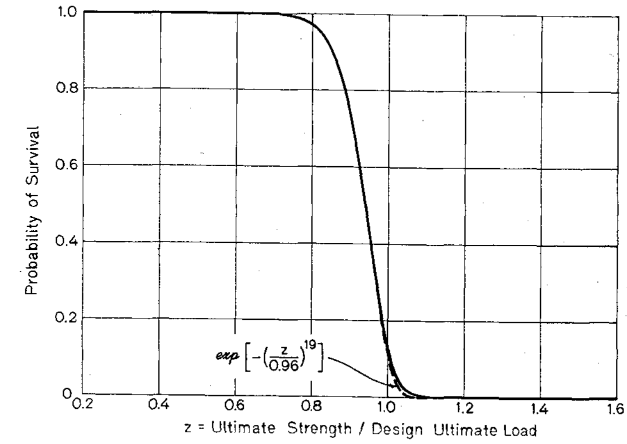

Fig. 3 Probability distribution of Z.

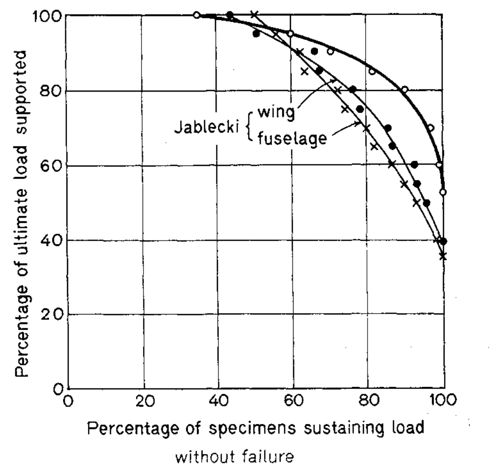

Fig. 4 Test results expressed in terms of design ultimate load (66 tests) compared with results of previous study (Ref. 1).

§ While this conclusion disregards the fact that strength deficiencies discovered in tests should be rectified or the design load reduced, no strength data of structural parts "retested" after strengthening have been supplied by the manufacturers.

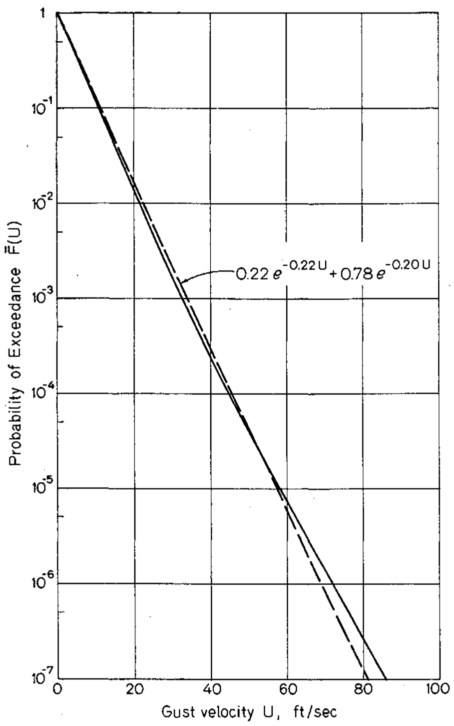

Fig. 5 Probability distribution of thunderstorm gust velocity.

during the 1940's. The curve obtained from current data indicates that the processes of design and construction of aircraft structures with respect to ultimate load failure have been improved within the two decades. For example, Fig. 4 suggests that about 90% of the currently produced aircraft structural members will sustain 80% of their nominal design ultimate load without failure, whereas in the 1940's only about 60% of the design ultimate load was sustained without failure by 90% of the structural members. However, there has been no significant change in the percentage of structural parts that fail to carry about 97% of the full nominal design ultimate load: only about one-half of all specimens tested can be expected to sustain this load level without failure.

3. Reliability Analysis for Thunderstorm Turbulence (Long-Range Aircraft)

A representative spectrum of thunderstorm gust velocity[4] forms a basis for the distribution function of extreme load intensities selected for the evaluation of the risk of failure and the determination of the reliability function for ultimate strength failure of aircraft structures. In Fig. 5, the probability of exceedance is chosen as ordinate instead of the cumulative frequency of load peaks per mile of flight. The spectrum in Fig. 5 (solid curve) is approximated by the equation

$$1 - F_U(u) = 0.22e^{-0.22u} + 0.78e^{-0.20u} \tag{11}$$

where U_{DU} is the specific thunderstorm gust velocity corresponding to the design ultimate load, and the ratio Λ, a random variable, represents the variable gust load in terms of the design ultimate load.

From Eq. (11), the distribution function of Λ is

$$F_\Lambda(\lambda) = 1 - \{0.22 \exp(-0.22\ U_{DU}\lambda) + 0.78 \exp(-0.20\ U_{DU}\lambda)\} \tag{12}$$

The reliability function $L_N(N)$ is defined as the probability of survival of the aircraft structures under a series of N load applications, so that

$$L_N(N) = \int_0^\infty [F_\Lambda(z)]^N f_Z(z) dz \tag{13}$$

For practical purposes, the following first approximation of Eq. (13) can be used.

$$L_N(N) = \exp(-Np_f) \text{ for } Np_f \ll 1 \tag{14}$$

where p_f is the probability of failure under single-load application obtained from

$$p_f = \int_0^\infty F_Z(\lambda) f_\Lambda(\lambda) d\lambda \tag{15}$$

From Eq. (10),

$$F_Z(\lambda) = 1 - L_Z(\lambda) = 1 - \exp[-(\lambda/0.96)^{19}] \tag{16}$$

and by differentiating Eq. (12) with respect to λ,

$$f_\Lambda(\lambda) = U_{DU}\{0.0484 \exp(-0.22\ U_{DU}\lambda) + 0.156 \exp(-0.20\ U_{DU}\lambda)\} \tag{17}$$

Substituting the previous expressions into Eq. (15),

$$p_f = U_{DU} \int_0^\infty \{1 - \exp[-(\lambda/0.96)^{19}]\} [0.484 \exp(-0.22 + U_{DU}\lambda) + 0.156 \exp(-0.20\ U_{DU}\lambda)] d\lambda \tag{18}$$

With the substitution $\omega = \exp(-\lambda)$, the previous integral is transformed into the form,

$$p_f = U_{DU} \int_0^1 \left\{1 - \exp\left[-\left(-\frac{\ln \omega}{0.96}\right)^{19}\right]\right\} \times [0.048\omega^{(0.22\ U_{DU} - 1.0)} + 0.156\omega^{(0.20\ U_{DU} - 1.0)}] d\omega \tag{19}$$

which is used for numerical calculation.

For various values of U_{DU}, p_f can be evaluated from Eq. (19), and the reliability function obtained from Eq. (14) as

Table 3 Data on ultimate strength of structures and components (in terms of design ultimate load or limit load)

Type of specimen	Type of loading	No. of tests
	Structures	
A-26B wings[a]	Bending	7
A3D-2P wings[a]	Bending	3
XA3D-1 wings	Bending, torsion	2
XA-2D1 wing	Bending, torsion	1
C-124A & C wing	Bending, torsion	1
XF-4D-1 wing	Bending	1
C-133A wing	Bending	1
A3D-2 wing	Bending, torsion	1
F51H wings[a]	Bending	3
C-130A, C-130B, C-130E wings[a]	Bending	10
C-130A fuselage cabin (nose section)[a]	Internal pressure	4
C-130A fuselage cabin (center section)	Internal pressure	1
C-130A fuselage cabin (aft section)	Internal pressure	1
T-38 horizontal stabilizer	Bending	2
B-70 spar-skin composite beam[a]	Compression	3
A3D-2 front-spar-fuselage frame	Bending, torsion	1
A3D-2 front-spar-frame element	Compression	1
C-130E, C-130B wing panels[a]	Compression	8
C-141A fuselage panels	Compression	3
C-141A fuselage panels	Shear	5
C-141A fuselage panels	Shear & compression	3
NB-66 wing cover assembly splices	Compression	4
		66

[a] Data contained in the plotting of Figs. 1 and 2.

a function of the number of gust load applications. Four values of U_{DU} have been assumed: U_{DU} = 90 fps, 75 fps, 60 fps, and 45 fps; the corresponding values of p_f and the function $L_N(N)$ are shown in Fig. 6.

The proportion of flight through thunderstorm turbulence is about 0.1% of flight distance.[5] It is assumed that 10 gusts/mile are encountered by an aircraft during thunderstorm flight at 10,000- to 20,000-ft level. Assuming further that the design life of the aircraft is 5×10^4 hr and the average flight velocity is 400 mph, the number of load applications is converted into time of flight in hour (4 gusts are equivalent to 1 hr of flight). The reliability function of ultimate strength of aircraft structures can, therefore, be expressed as af unction of flight hours as shown in Fig. 6.

In a recent report[6] the hours to reach or exceed ultimate strength have been computed for two aircraft designed by Lockheed: for the L-188 this figure is 7.14×10^7 hr and for the L-749: 2.38×10^8 hr. With these data, the reliability functions for these two aircraft have been plotted in Fig. 6. The difference between the present analysis and the Lockheed analysis is probably due to the fact that the latter is apparently involved only with the random character of the applied load while the strength is assumed constant, while in the present analysis the statistical character of strength is combined with that of gust load. The returns period of failure according to this analysis are 2.42×10^6 hr for U_{DU} = 90 fps, 2.02×10^5 hr for U_{DU} = 75 fps, 1.52×10^4 hr for U_{DU} = 60 fps, and 1.06×10^3 hr for U_{DU} = 45 fps.

4. Reliability Analysis for Operational (Maneuver) Loads (Fighter Aircraft)

A representative spectrum of operational (maneuver) load factors has been constructed on the basis of flight records from the F-105B and F-106A aircraft.[7] In Fig. 7, the probability of exceedance of the load factor n has been plotted for both aircraft and an intermediate spectrum selected for the reliability analysis which can be approximated by the equation

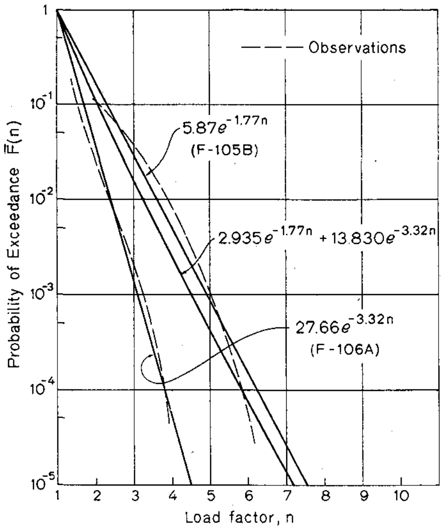

Fig. 7 Probability distributions of flight (maneuver) load factor.

$$\bar{F}_n(n) = 2.935e^{-1.77n} + 13.830e^{-3.32n} \quad (20)$$

where $\bar{F}_n(n) = 1 - F_n(n)$.

The assumption is now made that, for purposes of design, the operational load is proportional to the load factor or

$$\text{operational load}/R_{DU} = \text{load factor}/n_{DU} = \Lambda \quad (21)$$

where n_{DU} is the design load factor of ultimate load design and the ratio Λ, a random variable, represents the variable maneuver load factor in terms of the design ultimate load.

From Eq. (20), the distribution function of Λ is obtained

$$F_\Lambda(\lambda) = 1 - \{2.935 \exp(-1.77\lambda n_{DU}) + 13.830 \exp(-3.32\lambda n_{DU})\} \quad (22)$$

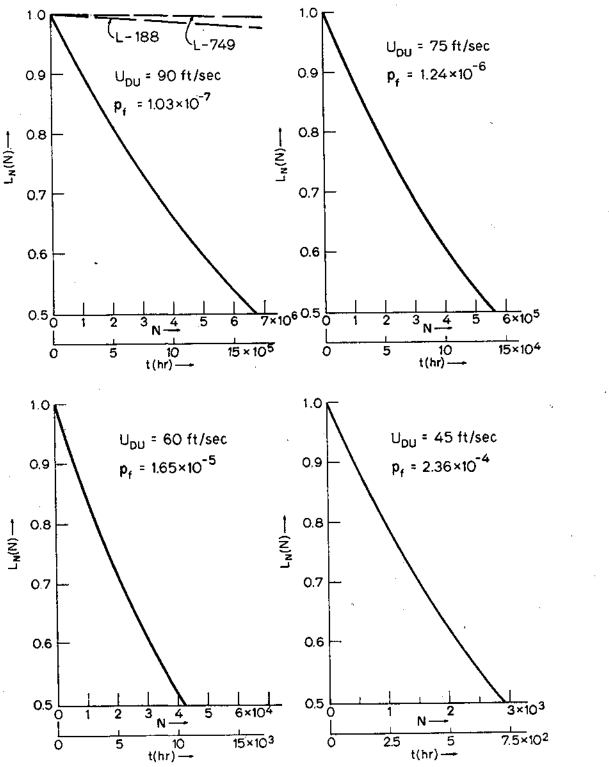

Fig. 6 Probability of survival under flight (thunderstorm) turbulence.

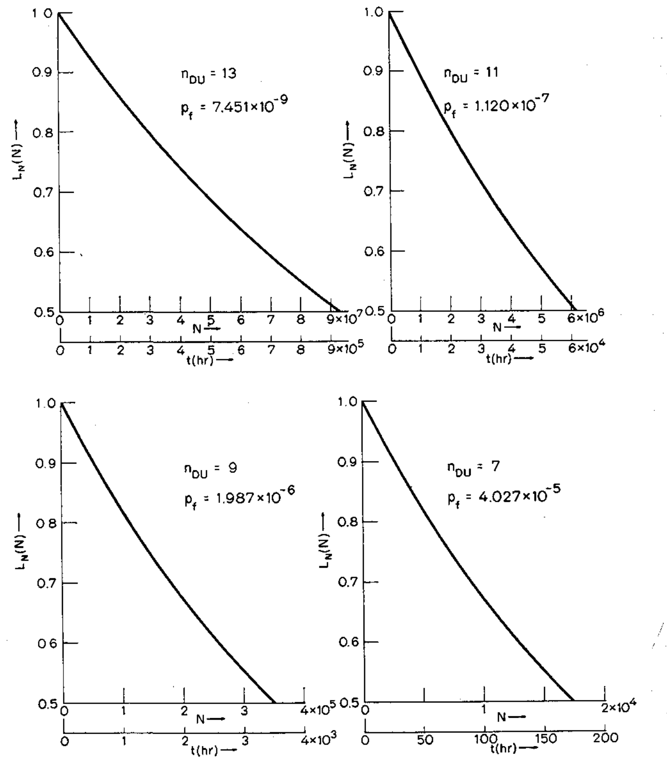

Fig. 8 Probability of survival under maneuver load factors.

and, therefore

$$f_\Lambda(\lambda) = n_{DU}[5.195 \exp(-1.77\lambda n_{DU}) + 45.916 \exp(-3.32\lambda n_{DU})] \quad (23)$$

The probability of failure according to Eq. (15) with $F_Z(\lambda)$ from Eq. (16)

$$p_f = n_{DU} \int_0^1 \left\{1 - \exp\left[-\left(-\frac{ln\omega}{0.96}\right)^{19}\right]\right\} \times [5.195\omega^{(1.77 n DU^{-1})} + 45.916\omega^{(3.32 n DU^{-1})}]d\omega \quad (24)$$

where $\omega = \exp(-\lambda)$.

Equation (24) has been numerically evaluated for the ultimate design load factors n_{DU} = 13, 11, 9, and 7 and the corresponding reliability functions have been constructed in accordance with Eq. (14) and are represented in Fig. 8 in terms of the number of load cycles and of hours of flight. The conversion is based on the assumption that roughly 10^2 load cycles are equivalent to 1 hr of flight at an average of 350–400 mph in accordance with the flight records.

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Response of Complex Structures to Turbulent Boundary Layers

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A turbulent boundary-layer loading function is developed for use with a finite-element structural analysis system and applied in a study of the random vibration of elastic aircraft structures. The method is demonstrated by computing the random vibration response of simple and complex structures; the method is evaluated by comparing computed and measured response on the simple panel. Deflection cross-power spectral density and deflection covariance are the criteria used to compare panel response. The simple panels are flat and clamped. The complex panels are flat and curved fuselage sections: 1) skin panels with tear straps and 2) six stringers attached to a skin strip. The effects of fuselage radius and of static in-plane loads due either to cabin pressurization or flight loads are discussed. The influence of boundary-layer thickness on the size of the region of coherent structural response is also discussed.

Nomenclature

$[A]$ = diagonal matrix of elemental areas on structure
A_i, A_j = elemental area on structure associated with i and j node points, in.^{2}
A_n = const, $A_1 = 12.0$, $A_2 = 7.20$, $A_3 = 1.58$
B = $1/0.8\delta^*$, in.$^{-1}$
$[C]$ = damping matrix
$C_{F_{ij}}(\omega)$ = force co-power spectral density (co-PSD) acting on plate pairs i and j, lb$^2\cdot$sec
$[C_F(\omega)]$ = force co-power spectral density matrix, lb$^2\cdot$sec
$[C_\delta(\omega)]$ = deflection co-power spectral desity matrix, in.$^2\cdot$sec
$[H(i\omega)]$ = complex frequency response matrix
$[K]$ = stiffness matrix
K^2 = normalization constant for power spectral density defined by $K^2 = \langle p^2 \rangle / \tau_w^2 = \sum_{n=1}^{3} A_n/K_n$
K_n = const, $K_1 = 13.9$, $K_2 = 2.94$, $K_3 = 0.471$
M = Mach number
$[M]$ = diagonal mass matrix
$Q_{F_{ij}}(\omega)$ = force quad-power spectral density acting on plate pairs i and j, lb$^2\cdot$sec
$[Q_F(\omega)]$ = force quad-power spectral density matrix, lb$^2\cdot$sec
$R(\ )$ = real part of a complex number, function, or matrix
S = $\omega\delta^*/U$, Strouhal number, dimensionless frequency
U = freestream airflow velocity or aircraft speed, in./sec
U_c = convection velocity, in./sec

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